Resonant Triple-Cyclers in the Jovian System

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Cyclers are space trajectories that can repeatedly encounter the same set of bodies indefinitely. Typically, cyclers are designed to encounter two bodies periodically, with only an occasional encounter with a third body. Because of the dynamics of the Laplace resonance in the Jupiter system, cycler trajectories that periodically return to three bodies are possible for Jupiter missions. Several cycler trajectories are proposed for purposes such as reducing mission length and increasing the number of flybys in a Jupiter system tour.

I. Introduction

Several authors have studied round-trip interplanetary missions that have led to the concept of cyclers.[1–6] One example of an interplanetary cycler mission that can travel between Earth and Mars indefinitely was designed by Byrnes, Longuski, and Aldrin.[7] These cyclers, known as Aldrin cyclers, have the capacity to encounter either Earth or Mars up to 15 times in a 15-year period with a total ΔV of under 2 km/s. Later research has also been done to apply low-thrust capabilities to cycler design in order to reduce the flyby V_∞ at Earth and Mars.[8,9]

Russell and Strange[10] extend the concept of cycler design from interplanetary cyclers to intermoon cycler trajectories. They focus on two-moon cyclers that would allow periodic return missions between any two of Jupiter's four Galilean moons and Titan-Enceladus cyclers in the Saturnian system. They propose a three-step model of designing these trajectories. They first find cycler trajectories in an ideal ballistic model assuming circular, coplanar orbits for the moons. They then optimize these ideal trajectories in a patched-conic ephemeris model to minimize ΔV. Finally, they integrate their patched-conic trajectories in a high-fidelity model.

We propose extending these two-moon cyclers to three-moon cyclers. We choose the Jupiter system for these proposed cyclers because all four of the Galilean moons have a high enough mass to permit gravity assist and the Galilean moons are all targets for scientific investigation. Additionally, the Laplace resonance among Io, Europa, and Ganymede[11–14] allows three-moon cyclers to have periods that are commensurate with the period of the Laplace resonance. Laplace-resonant cyclers reoccur indefinitely, so an entire Jupiter tour can be done while the spacecraft is in an Io-Europa-Ganymede cycler. In addition to indefinitely repeatable cyclers, we also propose cyclers that only reoccur once. These one-period three-moon cyclers contain two sets of three subsequent flybys and are useful for quickly reducing the orbital energy of a spacecraft in Jupiter orbit. These Laplace-resonant triple cyclers are found in an ideal model by using an extension of the Laplace resonance phase angles analysis proposed by Lynam et al.[11] The existence of these triple cyclers are confirmed by using the ephemeris models within AGI's STK[15] and JPL's MALTO.[16]

II. Laplace Resonance Phase Angles Analysis

A. Patched Conic Model

We perform the initial design of intermoon triple cyclers within the patched conic model.[17–19] This preliminary model does not incorporate the ephemerides of Jupiter's moons (other than constraining them to be consistent with the Laplace resonance[12]). Because the model is ephemeris-free, the spacecraft is modeled to perform a flyby whenever it passes through the orbit of a Galilean moon that is selected as a gravity-assist body. The true anomaly of the spacecraft (both before and after each flyby) and the time of flight of intermoon transfers for the Laplace-resonance phase-angle analysis required to find these intermoon triple cyclers are as follows:

$$f\text{moon} = -\cos^{-1}\left[\frac{-a_{moon} + a_{sc,in}(1-(e_{sc,in})^2)}{a_{moon}e_{sc,in}}\right] \tag{1}$$

where f_{moon} is the true anomaly of the spacecraft as it approaches the desired moon, a_{moon} is the semi-major axis of the desired moon's orbit, and $a_{sc,in}$ and $e_{sc,in}$ are the semi-major axis and eccentricity, respectively, of the spacecraft's

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orbit before the gravity-assist is modeled, and the time of flight between flybys is:

$$T_{1,2} = \sqrt{(a_{sc,out})^3/\mu_{\mathbf{J}}}\,[(E_{moon,2} - e_{sc,out}\sin E_{moon,2}) - (E_{moon,1} - e_{sc,out}\sin E_{moon,1})] \tag{2}$$

where

$$E_{moon} = -\cos^{-1}\left[(a_{sc,out} - a_{moon})/(a_{sc,out}e_{sc,out})\right] \tag{3}$$

and where E_{moon} is the eccentric anomaly of a moon; $a_{sc,out}$ and $e_{sc,out}$ refer to the semi-major axis and eccentricity of the spacecraft during its transfer. We note that Eq. 1 is the conic equation and Eq. 2 is Kepler's equation. The patched-conic model also requires that the orbital elements of an initial orbit are chosen and that the effects of all gravity-assists on the orbit are modeled.

B. The Laplace Resonance

The Laplace resonance is a 1:2:4 orbital resonance that governs the motion Ganymede, Europa, and Io. (I.e. Ganymede has double the orbital period of Europa and four times the orbital period of Io.) The Laplace resonance constrains the relative positions of Ganymede, Europa, and Io with the following equation:[11–14]

$$180^\circ = 2\lambda_{Ga} - 3\lambda_{Eu} + \lambda_{Io} \tag{4}$$

where λ_{Io} is Io's mean longitude, λ_{Eu} is Europa's mean longitude, and λ_{Ga} is Ganymede's mean longitude.

Lynam et al.[11] manipulated Eq. 4 to form the following phase angle relations among Ganymede, Europa, and Io:

$$\Delta\lambda_{Eu,Io} = 180^\circ + 2\Delta\lambda_{Ga,Eu} \tag{5}$$

$$\Delta\lambda_{Eu,Io} = \{\pm 60^\circ, 180^\circ\} + 2\Delta\lambda_{Ga,Io}/3 \tag{6}$$

$$\Delta\lambda_{Ga,Io} = \pm 90^\circ + 3\Delta\lambda_{Eu,Io}/2 \tag{7}$$

$$\Delta\lambda_{Ga,Io} = 3\Delta\lambda_{Ga,Eu} + 180^\circ \tag{8}$$

$$\Delta\lambda_{Ga,Eu} = \pm 90^\circ + \Delta\lambda_{Eu,Io}/2 \tag{9}$$

$$\Delta\lambda_{Ga,Eu} = \{\pm 60^\circ, 180^\circ\} + \Delta\lambda_{Ga,Io}/3 \tag{10}$$

where $\Delta\lambda_{Eu,Io}$ is the angle between the position of Europa and Io defined by the following relation:

$$\Delta\lambda_{Eu,Io} \equiv \lambda_{Io} - \lambda_{Eu} \tag{11}$$

(We note that $\Delta\lambda_{Ga,Io}$ and $\Delta\lambda_{Ga,Eu}$ are defined analogously.) The phase angle relations in Eqs. 5-10 allow the position (or two or three possible positions) of any one of the moons to be determined from the positions of the other two moons at any given time. Two of the equations (Eqs. 5 and 8) have unique solutions, two of the equations (Eqs. 7 and 9) have two solutions, and two of the equations (Eqs. 6 and 10) have three solutions. The physical interpretation of the multiple solutions is that a moon could be in two or three possible positions if only the angle between the other two moons is known. Lynam et al.[11] apply this phase angle analysis to find Laplace-resonant triple-satellite-aided capture sequences that capture into orbit around Jupiter using gravity-assist flybys of Ganymede, Europa, and Io. In this paper, we use Eqs. 5-10 to find similar triple-gravity-assist sequences that repeat periodically once the spacecraft is already in orbit.

The dynamics of the Laplace resonance are useful for finding these triple-gravity-assist sequences that can form triple cyclers. The mean motion of each moon govern its dynamics by varying the mean longitudes over time. We write the mean longitudes of each moon as a function of time. After one full period of the Laplace resonance (7.0509 days), the angles can be reset to their initial values:

$$\lambda_{Io}(t) = \lambda_{Io,0} + n_{Io}t \tag{12}$$

where $\lambda_{Io}(t)$ is the mean longitude of Io at time t; and $\lambda_{Io,0}$ is an arbitrary initial condition for the mean longitude of Io. We determine the phase angles between each set of moon over time, by finding the differences between Eq. 12 and similar equations for Europa and Ganymede. These time-varying phase angles can also be written in terms of the initial phase angles and the differences in the mean motions of the moons:

$$\Delta\lambda_{Eu,Io}(t) = \lambda_{Io}(t) - \lambda_{Eu}(t) = \Delta\lambda_{Eu0,Io0} + (n_{Io} - n_{Eu})t \tag{13}$$

where $\Delta\lambda_{Eu,Io}(t)$ is the time-varying phase angle between Europa and Io, $\Delta\lambda_{Eu0,Io0}$ is the initial phase angle between Europa and Io, and t is the time elapsed after the initial phase angle occurs.

Equation 13 and its analogous equations that relate the phase angles of the other sets of moons must satisfy the Laplace resonance. If a transfer between any two moons is known, the required position of the third moon can be determined from the dynamics of the Laplace resonance (Eqs. 12 and 13) and the Laplace resonance phase angle relations (Eqs. 5-10). We apply this technique to the Laplace triple cyclers by determining the required position of the third moon after the first two moon flybys and analyzing if the trajectory passes near it. Once the third flyby aligns with the trajectory, we use the patched-conic method to propagate the trajectory until it does another flyby of the first moon exactly one Laplace resonance period (7.055 days) later. Using the equations derived in this section, the time constaints are given for a Europa-perijove-Io-Ganymede triple cycler in terms of the true anomalies of each encounter and a single phase angle at the time of the initial flyby as follows:

$$n\,(T_{Lap}) = T_{Eu,Io} + T_{Io,Ga} + T_{Ga,Eu} \tag{14}$$

where n is a positive integer and T_{Lap} is the time required for Ganymede, Europa, and Io to complete one cycle of the Laplace resonance and

$$T_{Eu,Io} = \frac{f_{Eu,out} - f_{Io,in} - 180^\circ - 2\Delta\lambda_{Ga1,Eu1}}{n_{Io}} \tag{15}$$

$$T_{Io,Ga} = \frac{f_{Io,out} - f_{Ga,in} + 180^\circ + 3\Delta\lambda_{Ga1,Eu1} + (n_{Io} - n_{Ga})T_{Eu,Io}}{n_{Ga}} \tag{16}$$

$$T_{Ga,Eu} = \frac{f_{Ga,out} - f_{Eu,in} - \Delta\lambda_{Ga1,Eu1} - (n_{Eu} - n_{Ga})(T_{Eu,Io} + T_{Io,Ga})}{n_{Eu}} \tag{17}$$

where $T_{Eu,Io}$ is the time of flight from the Europa flyby through the trajectory's perijove to the Io flyby, $T_{Io,Ga}$ is the time of flight from the Io flyby to the Ganymede flyby, $T_{Ga,Eu}$ is the time of flight from the Ganymede flyby through the trajectory's apojove to the Europa flyby of the next cycle, $f_{Eu,out}$ is the outgoing true anomaly of the first Europa flyby, $f_{Io,in}$ is the incoming true anomaly of the Io flyby, $\Delta\lambda_{Ga1,Eu1}$ is the phase angle between Ganymede and Europa at the time of the first Europa flyby that is used to uniquely parameterize the problem, and n_{Io} is the mean motion of Io. The other variables used in Eqs. 15–17 are defined analogously. (Note that if other phase angles were used, the problem would not be uniquely parameterized because some of the equations would have two or three solutions.)

Another constraint on Laplace-resonant triple cyclers is that the Laplace resonance does not have exactly the same period as Ganymede's orbit (7.05 days vs 7.15 days). This lack of periodicity requires that the argument of perijove of the triple cycler's trajectory precess by about 5.2 degrees every Laplace resonance period in order to align the first set of three flybys with the second set of three flybys. Keplerian orbits have constant arguments of perijove, so the 5.2-degree offset must be corrected using the gravity-assist effect of all three flybys. Thus, triple cyclers cannot be found using the ideal model that Russell and Strange[10] used to find double cyclers because that model only modeled one gravity-assist per cycle. Furthermore, in order to repeat indefinitely, the triple-cyclers must have the same orbital periods and perijoves for every cycle. In order to meet all of these constraints, we adjust the flyby altitudes of each of the three flybys.

III. Single-period, Laplace-resonant Triple Cyclers

The solution of Eq.14 where $n = 1$ describes a single-period, Laplace-resonant triple cycler. A spacecraft on this cycler trajectory would visit Io, Europa, and Ganymede once per Laplace resonance period. Figure 1 shows a triple cycler that begins at Europa, travels through its perijove to Io, transfers to Ganymede, and travels through its apojove to reach Europa 7.05 days later (with a new perijove that is 5.2 degrees different from its original perijove). This cycler also retains the same orbital period and perijove for both cycles. Unfortunately, as noted in Table 1, the cycler requires a subsurface flyby (at -175 km) of Europa in order to satisfy those constraints. Because ΔV is required to correct this flyby-altitude constraint violation, the single-period, Laplace-resonant triple cycler is not a ballistic cycler.

A. Patched-conic Ephemeris Model

The patched-conic, ephemeris-free model used to generate Fig. 1 did not give a ballistic solution for a single-period, Laplace-resonant triple cycler, so a more advanced model is needed to optimize the ΔV of a triple cycler. We use JPL's MALTO[16] software to minimize the ΔV required to execute one of these cycler orbits. Due to computational difficulties in generating these cyclers, we only optimize a triple cycler for two cycles. In this case, a Ganymede-Io-perijove-Europa cycler is used, which is essentially equivalent to a Europa-perijove-Io-Ganymede cycler. Because a

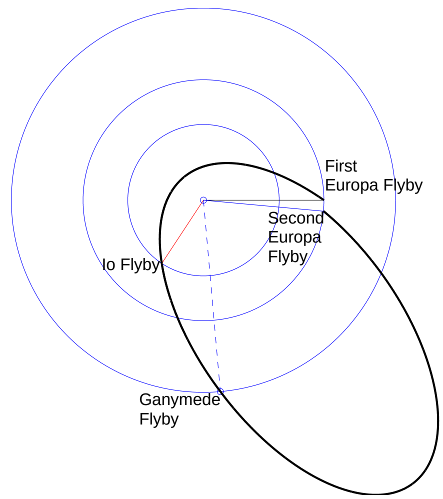

Figure 1. A Europa-perijove-Io-Ganymede, single-period, Laplace-resonant triple cycler. The spacecraft completes three flybys in the first cycle then starts the second cycle with a flyby of Europa.

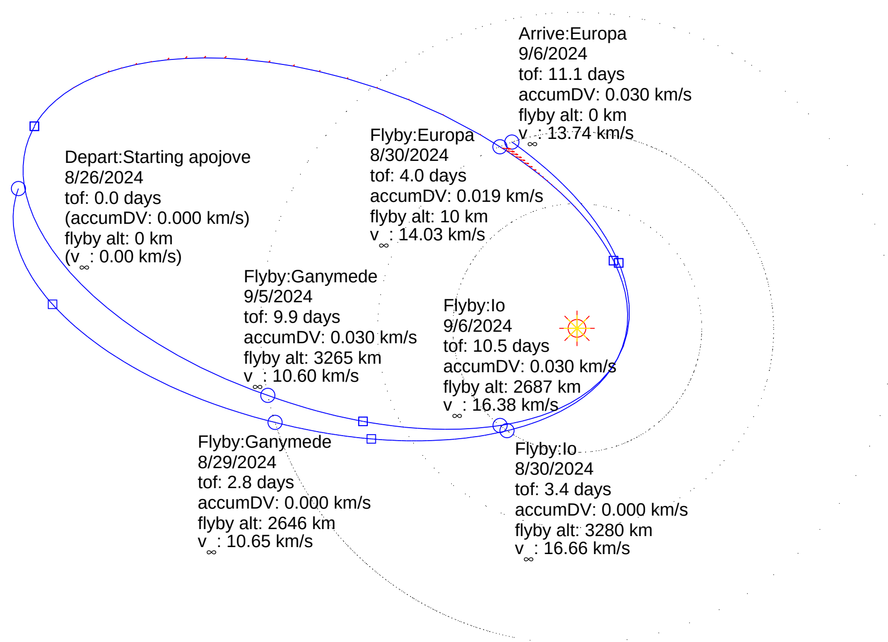

Figure 2. A Ganymede-Io-perijove-Europa, single-period, Laplace-resonant triple cycler optimized using MALTO. The spacecraft completes three flybys in each of the first two cycles.

Table 1. Orbital Parameters for a Europa-Perijove-Io-Ganymede Triple Cycler

Parameter	Value
Orbit Period	7.055 days
Perijove	2.3588 km
Europa flyby altitude	-175 km
Io flyby altitude	2500 km
Ganymede flyby altitude	10000 km

subsurface Europa flyby was indicated by the preliminary analysis, we set the minimum flyby altitude for the Europa flyby to be as low as possible (10 kilometers) in order to minimize ΔV without crashing into Europa's surface. Constraints were added that forced the two encounters of each moon to occur exactly 7.05 days (i.e. one Laplace resonance period) apart. These constraints force the orbital parameters of each cycle to be nearly equivalent. The maximum accumulated ΔV is set to be as small as feasible, so that the total ΔV is as close to the global minimum as possible. Figure 2 shows the converged MALTO trajectory for two cycles of a single-period, Laplace-resonant triple cycler. The total required ΔV for this trajectory is 30 m/s, so we conjecture that a spacecraft could stay in this orbit for more than two cyclers at a cost of approximately 30 m/s per cycle.

The advantage of using MALTO is that it adjusts the flyby parameters, trajectories, and maneuvers as needed to converge on a solution. The converged solution automatically has many of the same characteristics as the ephemeris-free solution (e.g. a 7.05 day orbital period for both cycles, a 5.2 degree change in argument of perijove between cycles, and three consecutive flybys in both cycles). We also note that most of the ΔV (shown as arrows in Fig. 2) is used immediately before the Europa flyby in order to prevent it from being a subsurface flyby as in the ephemeris-free solution. The Europa flyby in the second cycle is the endpoint of the entire trajectory, so it does not require a flyby altitude constraint. If the third cycle of the trajectory is needed, the second Europa flyby would also require additional ΔV to prevent it from being a subsurface flyby.

IV. Multiple-period, Laplace-resonant Triple Cyclers

In addition to single-period triple cyclers, which have an orbital period that is commensurate with the Laplace resonance, we investigate the possibility of multiple-period triple cyclers whose periods correspond to the $n < 1$ solutions of Eq. 14. Single-period triple cyclers could not repeat ballistically because a subsurface Europa flyby was necessary to implement the required 5.2 degree precession of the perijove of the cycler. Since a double-period triple cycler would need to precess the spacecraft's perijove by 10.4 degrees, it would require an infeasible amount of propellant to maintain. The precession problem would become even worse for triple cyclers with orbital periods that were three or more times the Laplace resonance period. However, the above analysis assumes that Ganymede, Europa, and Io must always be encountered in the same order. If we relax that assumption, new triple-cycler trajectories emerge that encounter the three moons in a different order each cycle.

A. Apojove Angles for Laplace-Resonant Triple Cyclers

Every multiple-period, Laplace-resonant triple cycler contains three flybys of Ganymede, Europa, and Io in rapid succession within one day of each other. (Navigational challenges are beyond the scope of this paper.) After these flybys, the cycler spacecraft spends the remainder of its orbital period transfering to its apojove (furthest distance from Jupiter) then returning to the vicinity of the three moons. The orbital period of an orbit of the triple cycler is dependent on the flyby order of the first and second sets of three flybys and the angular distance that the Laplace resonance precesses during the orbit. In order to quantify this dependence, we perform a further phase angle analysis of the Laplace resonance.

Only one inferior conjunction between Ganymede and Europa occurs during a Laplace resonance period, so the angular position of Ganymede and Europa during conjunction is a good reference angle to measure the position of the apojove of one of the cycles. The four possible flyby orders for a triple cycler are: Ganymede-Io-perijove-Europa, Ganymede-Europa-perijove-Io, Europa-perijove-Io-Ganymede, and Io-perijove-Europa-Ganymede. Each of these flyby orders produce unique apojoves that are at different angles from the local Ganymede-Europa conjunction angle. These angles are calculated for each flyby order as follows. First, we calculate the phase angle between Ganymede and Europa at the time of the final flyby in a cycle, $\Delta\lambda_{Ga3,Eu3}$, using Eqs. 5–10. Next, we apply Eq. 13 to the Ganymede-Europa phase angle with the initial phase angle being set to zero to correspond with a Ganymede-Europa conjunction.

$$\Delta\lambda_{Ga3,Eu3} = (n_{Eu} - n_{Ga})T_3 \tag{18}$$

where $\Delta\lambda_{Ga3,Eu3}$ is the phase angle between Ganymede and Europa at the time of the last flyby, n_{Eu} is the mean motion of Europa, and T_3 is the time between the local Ganymede-Europa conjunction and the time of the last flyby. The angle between the final flyby and the apojove of the orbit can be calculated by using Eq. 1 to find the true anomaly of the final moon encounter.

$$\Delta f = 180° - f_{moon,3} \tag{19}$$

where Δf is the change in true anomaly between the apojove and the final moon encounter and $f_{moon,3}$ is the true anomaly of the final moon encounter. Equations 12, 18, and 19 are combined to form equations for the angle between the apojoves and the Europa-Ganymede conjunction angle of each cycle. For cycles ending with Ganymede or Europa, this angle is:

$$\Delta f_{apo,GEconj} = (180° - f_{moon,3}) - \frac{n_{moon}\Delta\lambda_{Ga3,Eu3}}{n_{Eu} - n_{Ga}} \tag{20}$$

where $\Delta f_{apo,GEconj}$ is the angle between the apojove of the orbit and the local Ganymede-Europa conjunction, n_{moon} is the mean motion of Ganymede or Europa, and $f_{moon,3}$ is the true anomaly of Ganymede or Europa (depending on whether Ganymede or Europa is the last flyby in a cycle). For the (one) cycle ending with Io, the angle becomes

$$\Delta f_{apo,GEconj} = (180° - f_{Io,3}) - \frac{n_{Eu}\Delta\lambda_{Ga3,Eu3}}{n_{Eu} - n_{Ga}} - \Delta\lambda_{Eu3,Io3} \tag{21}$$

where $f_{Io,3}$ is the true anomaly of Io and $\Delta\lambda_{Eu3,Io3}$ is the phase angle between Europa and Io at the time of the Io flyby. These equations are used along with previous phase angle analysis done by Lynam et al.[11] to find the apojove angles for all four flyby orders. Figure 3 depicts the angles between the apojoves and the local Ganymede-Europa conjunctions.

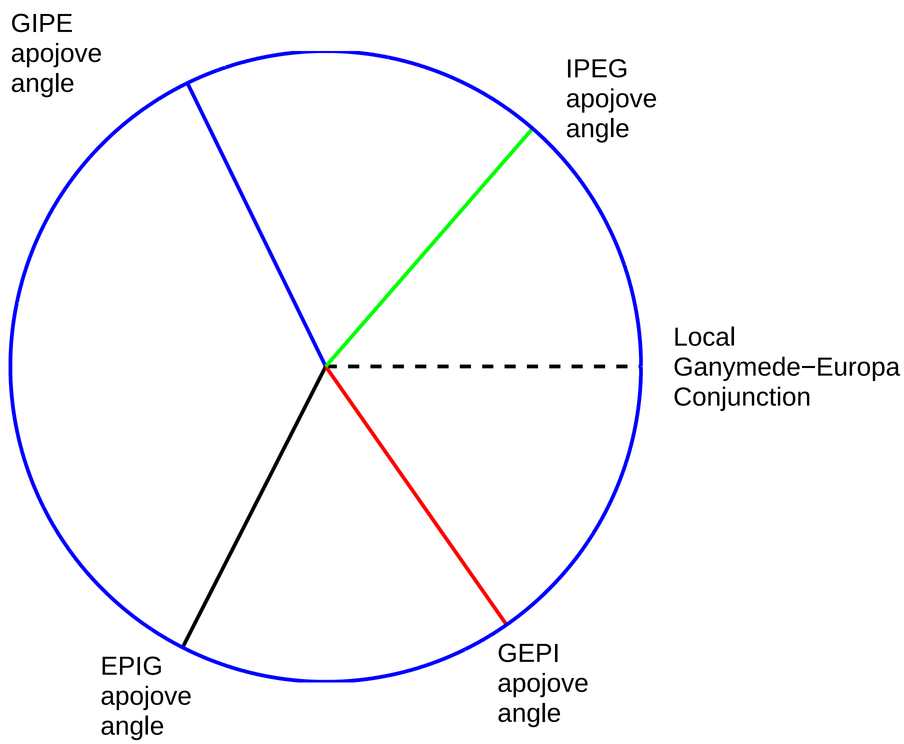

Figure 3. A plot of the angles between the local Ganymede-Europa conjunction angle and the apojoves of several possible triple cycler flyby orders. (Note that the angles precess counter-clockwise at a rate of 5.2 degrees per week.)

The apojove angles in Fig. 3 precess by 5.2 degrees every Laplace resonance period. Thus, transfers can be formed between different cycles by finding the amount of time that it takes for one apojove angle to reach another. This time fixes the required period of the transfer orbit between the two cycles. We divide the angular difference between the apojoves in Fig. 3 by 5.2 degrees and multiply the result by 7.05 days to estimate the transfer orbit period. Table 2 shows the orbital transfer periods between the various cycles. The apojove angles precess counter-clockwise, so each transfer between cycles occurs in a single direction (e.g. a GIPE [Ganymede-Io-perijove-Europa] cycle precedes a transfer to a EPIG [Europa-perijove-Io-Ganymede] cycle, but not vice versa). Additionally, the perijoves of each cycle are different, so a maneuver is required at the apojove of each orbit to correct the differences.

Table 2. Orbit Periods for Transfers between Cycles of Three Moons

First Cycle	Second Cycle	Orbit Period (days)	Perijove after Second Cycle (R_J)	Apojove ΔV (m/s)
GIPE	EPIG	170	1.77	10
EPIG	GEPI	84	1.10	55
GEPI	IPEG	140	0.91	15
IPEG	GIPE	91	2.08	240

A multiple-period, Laplace-resonant triple cycler would continuously cycle among the four flyby sequences in order to flyby Ganymede, Europa, and Io during each cycle. Unfortunately, the apojove ΔV required to align the cycles would become prohibitively expensive over a few years in this orbit. However, the orbital periods are much greater than the single-period triple cyclers, so the amount of ΔV required for a given period of time would be much lower. It may be advantageous for a mission to only perform two or three orbits of this cycler then continue on a conventional Jupiter tour with no more triple cycler orbits.

B. Integrated Multiple-period, Laplace-resonant Triple Cyclers

Laplace resonance phase angle analysis is an analytical method that estimates various parameters of multiple-period triple cyclers. In order to perform a higher-fidelity analysis, we integrate two cycles of a multiple-period, Laplace-resonant triple cycler in STK.[15] The first cycle is actually a triple-satellite-aided capture sequence (Ganymede-Io-JOI-Europa) similar to those developed by Lynam et al.[11] After this capture sequence, the spacecraft orbits Jupiter for about 177 days before it performs the second cycle (Europa-perijove-Io-Ganymede). These two cycles are integrated with the same high-fidelity propagator used by Lynam et al. This propagator includes the point-mass gravity of the Sun, Jupiter, and the four Galilean moons; solar radiation pressure; the oblateness of Jupiter and the Galilean moons; and general relativity perturbations.

The STK integration required two maneuvers to patch the two cycles together. Because these trajectories in STK were integrated but not optimized, we do not claim that these maneuvers use the minimal amount of ΔV to patch the two cycles together. It is possible that these cycles could be patched together with a trajectory optimizer to find a more optimal solution. The six moon flyby and the two maneuvers are detailed in Table 3. The total (non-optimal) ΔV for this cycler is 63 m/s, which is considerably greater than the 10 m/s estimate from Table 2.

These two cycles of triple flybys allow a spacecraft to capture into orbit about Jupiter, orbit Jupiter for 177 days, and then reduce the period of its second orbit to 60 days. A mission using this trajectory would have more scientific opportunities to study Ganymede, Europa, and Io than a single-satellite-aided capture sequence that only uses single flybys to reduce the orbital period of the spacecraft.

Table 3. STK Integration of Two Cycles of a Multiple-period, Laplace-resonant Triple-cycler

Flyby / Maneuver	Event Time (UTC)	Flyby Altitude (km) / ΔV (m/s)
Ganymede	29 Aug 2024 19:22:13	300.8
Io	30 Aug 2024 05:52:52	357.1
Europa	30 Aug 2024 18:22:04	301.6
Maneuver 1	19 Oct 2024 18:22:04	22.9
Manuever 2	11 Feb 2025 21:36:00	41.1
Europa	21 Feb 2025 21:36:00	282.6
Io	22 Feb 2025 10:21:29	54.5
Ganymede	22 Feb 2025 21:47:41	292.4

V. Discussion

Laplace-resonant triple cyclers extend the concept of cyclers to include flybys of three of Jupiter's Galilean moons every cycle. These cyclers offer Jupiter tour designers the flexibility to create missions that can periodically monitor Ganymede, Europa, and Io. The single-period, Laplace-resonant cyclers can re-encounter Ganymede, Europa, and Io once every week and would be useful for Jupiter system tours that would require a large number of flybys during a short mission duration. Additionally, these trajectories accumulate large amounts of radiation due to their close proximity to Jupiter and would be somewhat difficult to navigate due to their large number of flybys over a short period of time. Although by the time these spacecraft would begin these cycler trajectories, the spacecraft's orbit would be determined precisely via radiometric and optical orbit determination. The sensitivity of triple-flyby sequences to small errors was estimated by Lynam et al.[11] Since the ΔV cost for these single-period triple cyclers is estimated about 30 m/s per week, a year-long triple-cycler tour would require 1.56 km/s of deterministic ΔV to execute.

Conversely, multiple-period, Laplace-resonant triple cyclers require much less ΔV, but only re-encounter Ganymede, Europa, and Io about once every 100 days. These cyclers would also be inside Jupiter's radiation environment for a much lower percentage of the total mission duration, and the navigational challenges would be more easily met during to the long period of time between cycles. The multiple-period triple cyclers would be good for missions that would survey Ganymede, Europa, and Io over a longer duration. Executing only two cycles of a multiple-period, Laplace-resonant triple cycler would be advantageous for capturing a spacecraft into orbit about Jupiter and quickly reducing its period. Additionally, a spacecraft could reduce its period using a multiple-period, Laplace-resonant triple cycler then use several non-cycler gravity assists to adjust its orbit into a single-period, Laplace-resonant triple cycler.

We design these cyclers in an idealized, ephemeris-free, patched conic model using a Laplace-resonance phase angle analysis. This analysis exploits the unique periodicity of the Laplace resonance among Ganymede, Europa, and Io to design repeating triple-cycler trajectories that would not be possible with any other set of three moons. Although this analysis provides a large amount of information about the nature of both single-period and multiple-period triple cyclers, it is too low-fidelity to confirm the existence of these trajectories. We validate this method for single-period, Laplace-resonant triple cyclers by optimizing two cycles that each encountered Ganymede, Europa, and Io in the same order, and the two cycles are separated by one Laplace resonance period (about a week). This optimization used JPL's MALTO software, which contains a patched-conic, ephemeris orbital model and a parameter optimizer.[16] MALTO was sufficient for this validation because the cycler remains close to Jupiter and is too short to be significantly perturbed. On the other hand, the multiple-period, Laplace-resonant triple cycler spends a large amount of time in a region where solar perturbations are significant, so STK[15] was used to integrate the triple-cycler trajectory in an ephemeris model.

VI. Conclusions and Future Work

We expanded the concept of a cycler to include spacecraft in Jupiter orbit that encounter three of Jupiter's Galilean moons in a cycle. Since Ganymede, Europa, and Io are locked in the Laplace resonance, many of the characteristics of two-moon or two-planet cyclers were extended to Laplace-resonant triple cyclers via a Laplace-resonance phase-angle analysis. One such cycler is termed the "single-period, Laplace-resonant triple cycler" because its orbital period is the same as the period of the Laplace resonance. This cycler has the capacity to perform one flyby of each of the three moons every seven days. Unfortunately, this cycler is not ballistic because it requires about 30 m/s of ΔV every seven days to maintain the trajectory. The best use of the single-period, Laplace-resonant cycler would be for missions that require large numbers of Ganymede, Europa, and Io flybys in a relatively short period of time. One example of this cycler was optimized in JPL's MALTO software, but there are several other similar cyclers that need to be optimized in order to more fully explore the mission design of single-period, Laplace-resonant triple cyclers.

Another cycler is called the "multiple-period, Laplace-resonant triple cycler" because its orbital period is between 12-24 Laplace resonance periods (84 to 170 days). This cycler is not ballistic either, but requires less total ΔV than the single-period triple cycler and still re-encounters each of the three moons periodically. The multiple-period triple cycler is more useful for reducing the orbital period of a spacecraft orbiting Jupiter or for observing Ganymede, Europa, and Io over an extended-length mission. In particular, we integrated a trajectory in STK that included a triple-satellite-aided capture sequence, a 177-day capture orbit, and another three-flyby sequence after the capture orbit. This integrated trajectory was not optimized, so another extension of this analysis would be to find optimal multiple-period, Laplace-resonant triple cyclers.

Laplace-resonant triple cyclers provide mission designers with another set of trajectories that can be incorporated inside Jupiter tours. These cyclers can help meet possible mission objectives, such as increasing the number of flybys in a tour or quickly decreasing the orbital period of a Jupiter orbiter. The Laplace-resonance phase angle analysis presented in this paper could also be used as a more general mission design technique for Jupiter tours involving flybys of Ganymede, Europa, and Io.

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